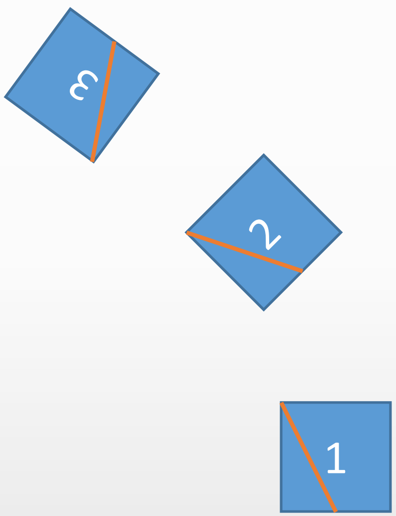
Kinematics & Dynamics of Linkages

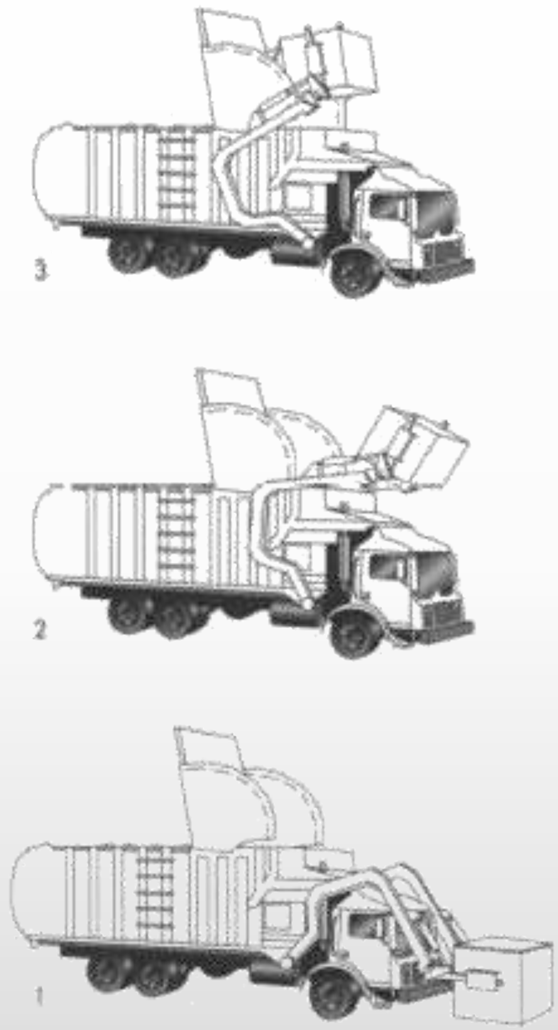
Lecture 10 – Geometric Linkage Synthesis

Chapter Objectives

1. How to design a linkage that delivers a desired output
2. Types of linkages synthesis
3. Limitation of linkages synthesis
4. Geometric synthesis

Example





Types of Synthesis

- **Path Generation**
  - Control of a **point** in the plane such that it follows some prescribed **path**
  - **No orientation** control to the link that contains the point of interest

- **Motion Generation**
  - Control of a **line** in the plane such that it assumes some sequential set of prescribed **positions**
  - **Orientation** of the link containing the line is important

Linkage Synthesis – Precision Points

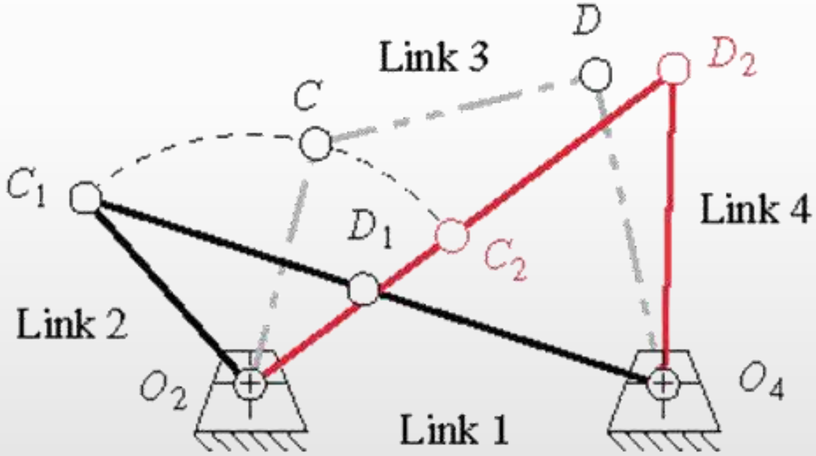
- The points prescribed for successive locations of the output link
- Limited by the number of equations available
- Four bar linkage may have up to 5 precision points
- Usually, we use 2 or 3 precision points
  - Reduces to a set of linear simultaneous equations
  - More than 3 points requires complex software
- The solutions provide no guarantee as to the location of a linkage between precision points.

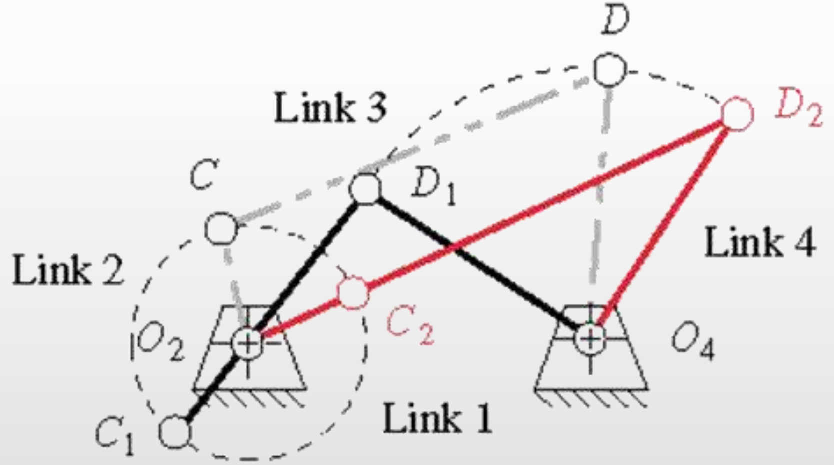
Linkage Synthesis – Verification

- Linkage synthesis procedures often provide solutions at the specified locations but not necessary in between these locations
- They say nothing about the linkage's behaviors between those positions
- It is possible that the resulting linkage will be incapable of moving from one precision point to another due to some constraint:
  - **Toggle position**
  - **Transmission angle**

Linkage Synthesis – Toggle Positions

You need to check that the linkage can reach all of the specified design positions without encountering a toggle position.



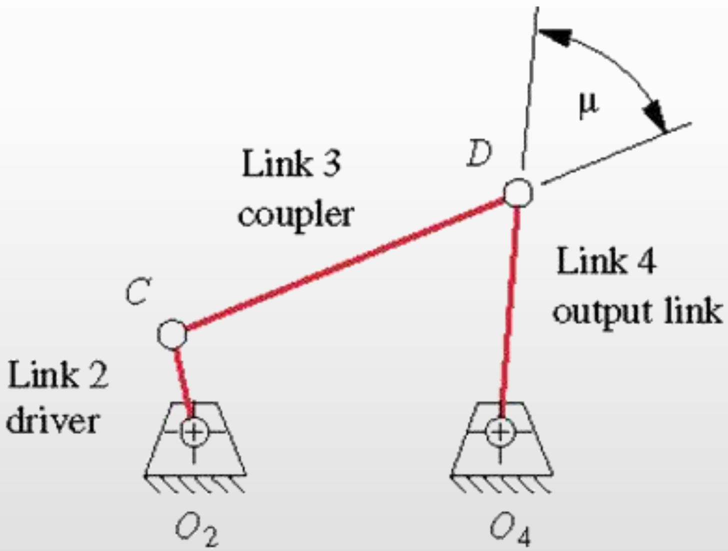


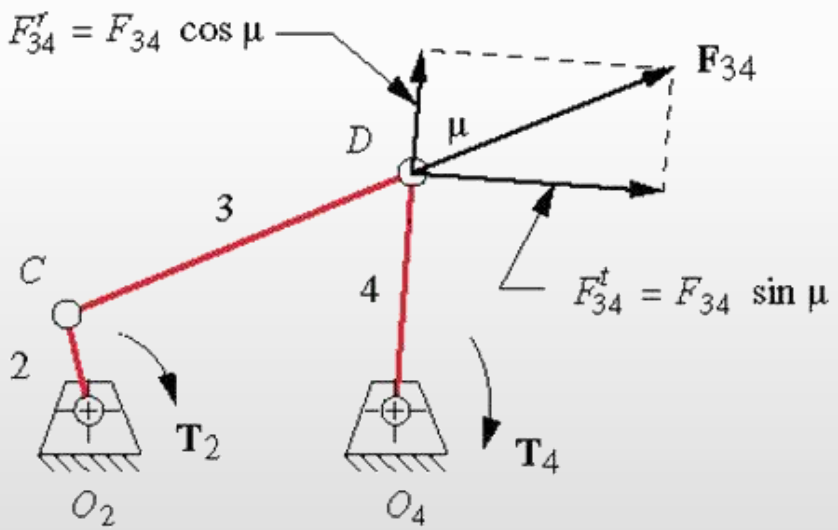
Non-Grashof: Triple rocker (Should be Avoided)

Grashof: Crank rocker

Linkage Synthesis – Transmission angle

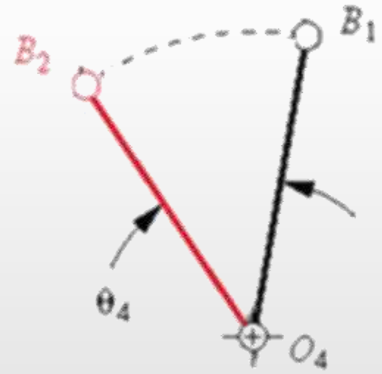
The angle between the output link and the coupler should be a minimum μ > 40º (optimum 90º)





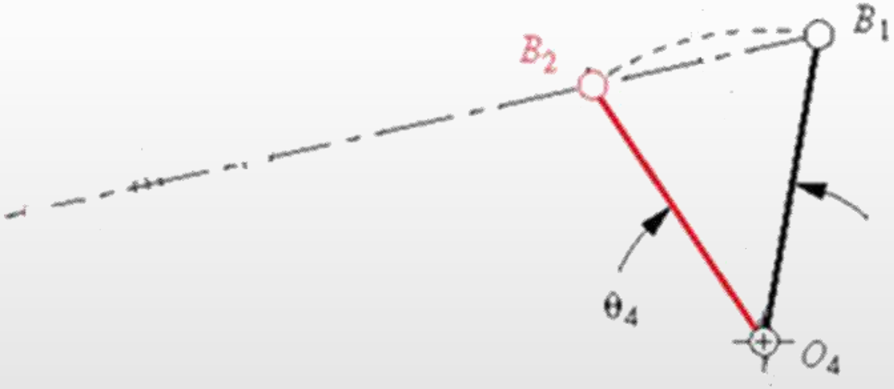
Geometric Linkage Synthesis – Case 1
2-Position Rocker Output

Design a four bar Grashof crank-rocker speed motor input to give 45° of rocker motion with equal time forward and back, from a constant speed motor input.



Geometric Linkage Synthesis – Case 1
2-Position Rocker Output – Solution: Step 1

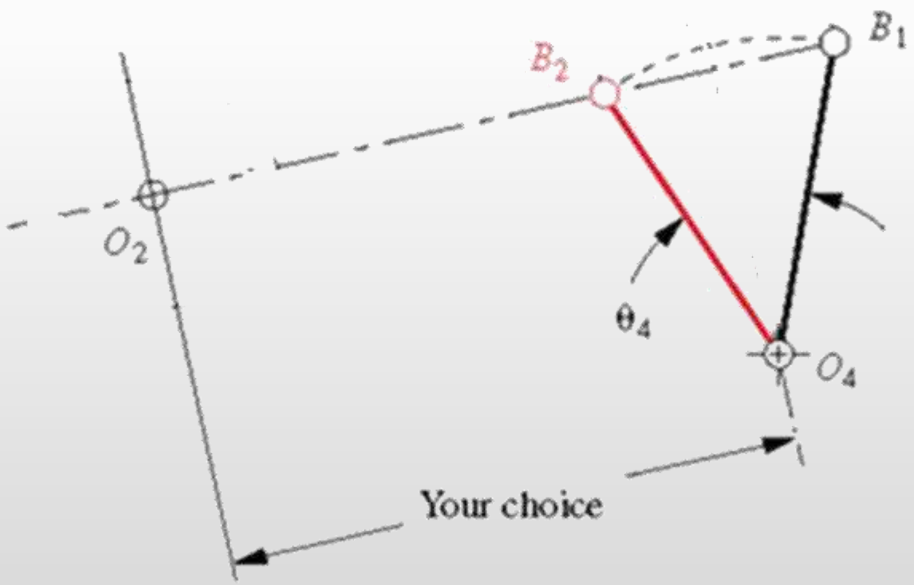
Draw the chord B_1B_2 and extended it in either direction.



Geometric Linkage Synthesis – Case 1

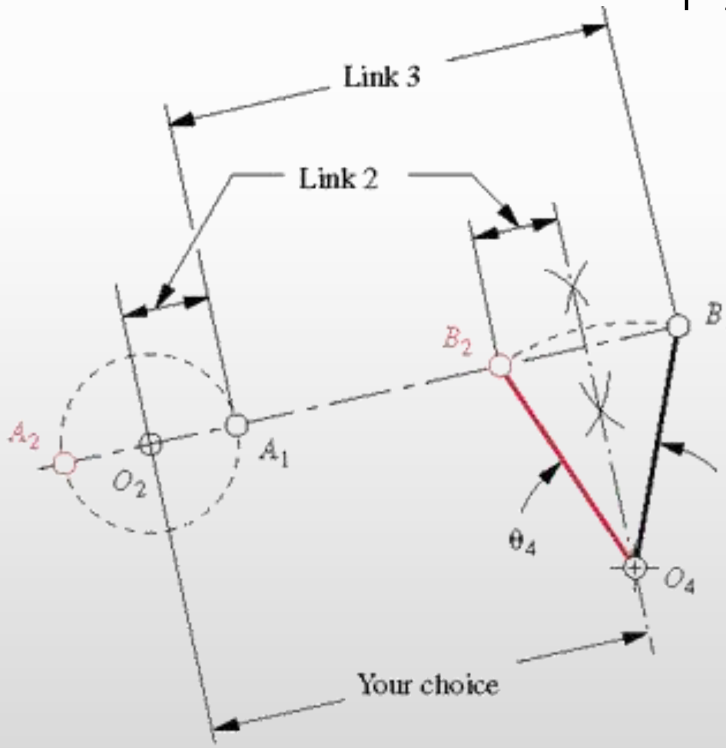
2-Position Rocker Output – Solution: Step 2

Select a convenient point O_2 on the line B_1B_2 extended.



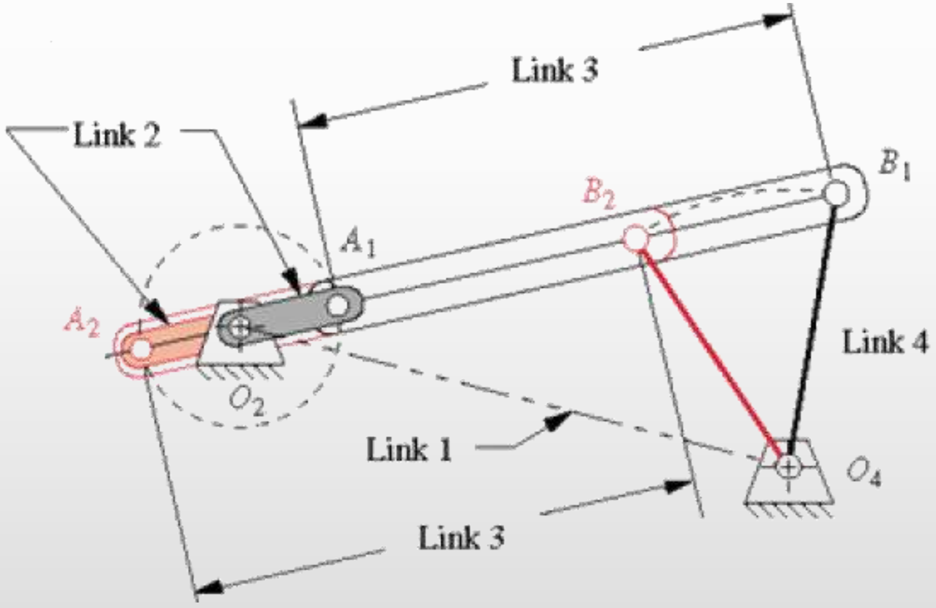
Geometric Linkage Synthesis – Case 1
2-Position Rocker Output – Solution: Step 3

Bisect line segment B_1B_2, draw a circle of that radius about O_2 and label the two intersection of the circle and B_1B_2 extended, A_1 and A_2



Geometric Linkage Synthesis – Case 1
2-Position Rocker Output – Solution: Step 4

Measure ground length 1, crank length 2, and rocker length 4.



Geometric Linkage Synthesis – Case 1

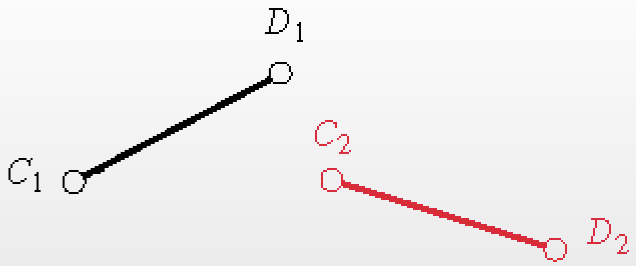
2-Position Rocker Output – Solution: Steps 5 and 6

Step 5: Check the Grashof condition and redo steps 2 to 5 with O_2 further from O_4 if non-Grashof.

Step 6: Build the linkage model and check its function and transmission angles.

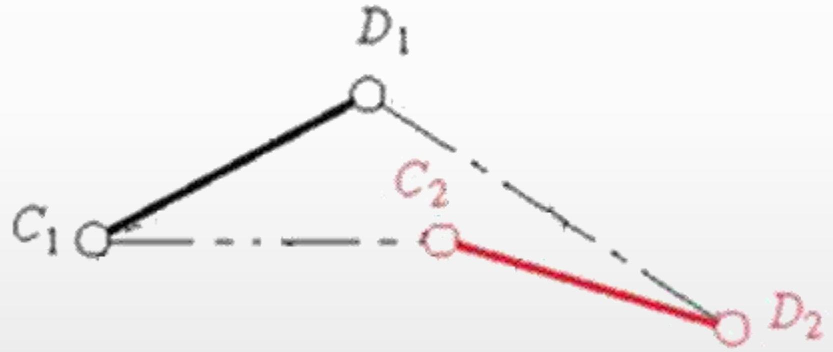
Geometric Linkage Synthesis – Case 2
2-Position Rocker Output Motion

- Rocker Output-Two Position with Complex Displacement (Motion)
- Design a four bar linkage to move link CD from C_1D_1 to C_2D_2.



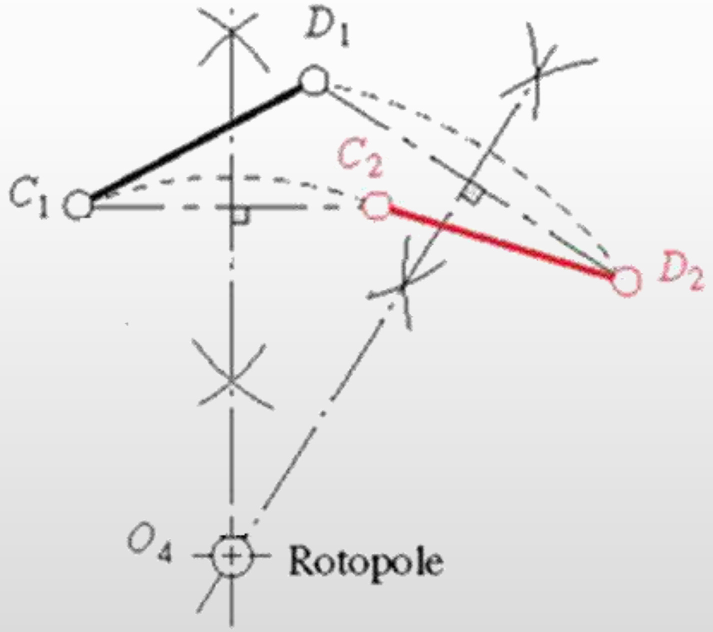
Geometric Linkage Synthesis – Case 2
2-Position Rocker Output Motion – Solution: Step 1

Draw construction line from point C_1 to C_2 and from point D_1 to D_2



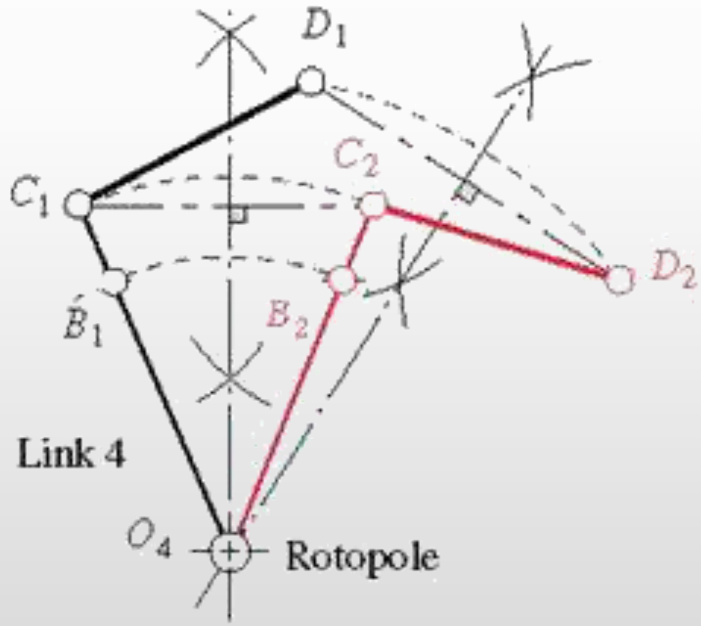
Geometric Linkage Synthesis – Case 2
2-Position Rocker Output Motion – Solution: Step 2

Bisect line C_1C_2 and line D_1D_2 and extend their perpendicular bisectors to intersect at O_4. Their intersection is the rotopole.



Geometric Linkage Synthesis – Case 2
2-Position Rocker Output Motion – Solution: Step 3

Select a convenient radius and draw an arc about the rotopole to intersect both lines O_4C_1 and O_4C_2. Label the intersection B_1 and B_2.

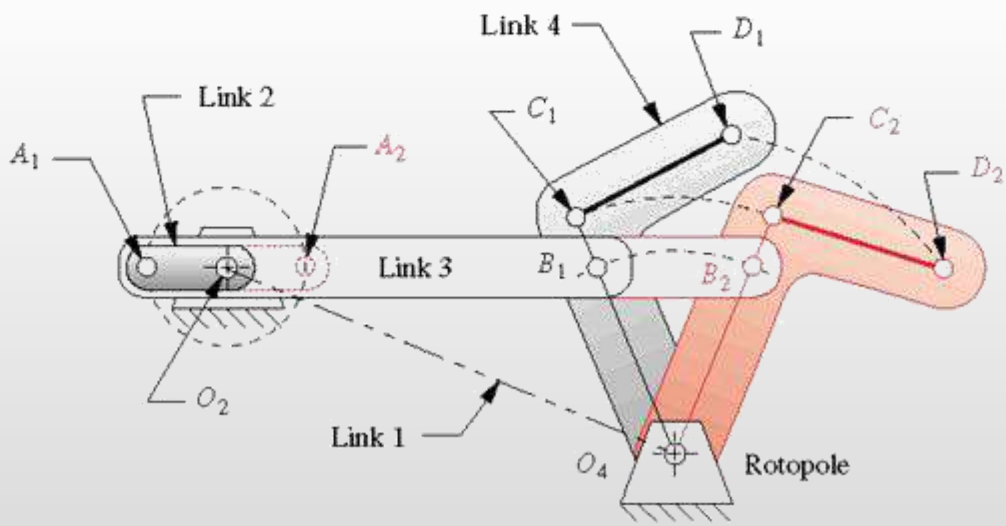


Geometric Linkage Synthesis – Case 2

2-Position Rocker Output Motion – Solution: Steps 4-6

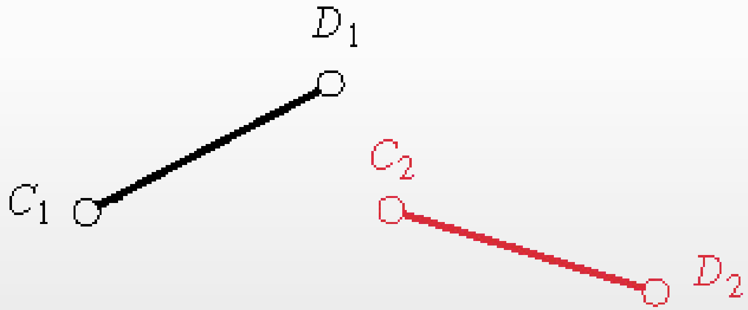
Steps 4-5: Repeat steps 4 and 5 of previous procedure

Step 6: Make a model of the linkage and articulate it to check its function and its transmission angles



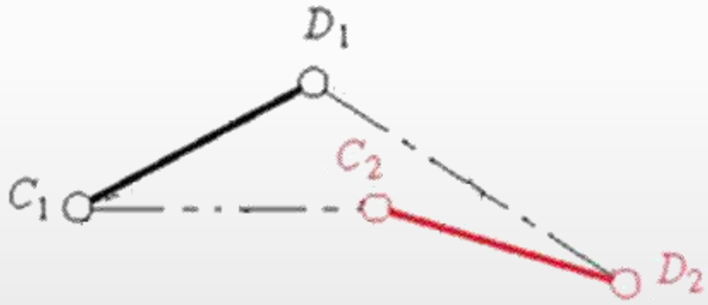
Geometric Linkage Synthesis – Case 3
2-Position Moving Pivots Motion

Design a fourbar linkage to move link CD from C_1D_1 to C_2D_2 (with moving pivots at C and D).



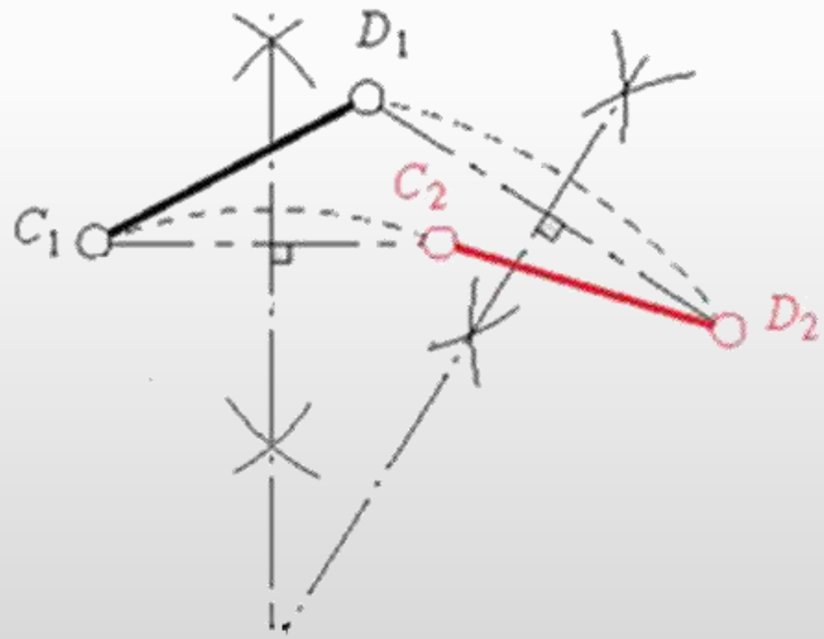
Geometric Linkage Synthesis – Case 3
2-Position Moving Pivots Motion – Step 1

Draw construction line from point C_1 to C_2 and from point D_1 to D_2.



Geometric Linkage Synthesis – Case 3
2-Position Moving Pivots Motion – Step 2

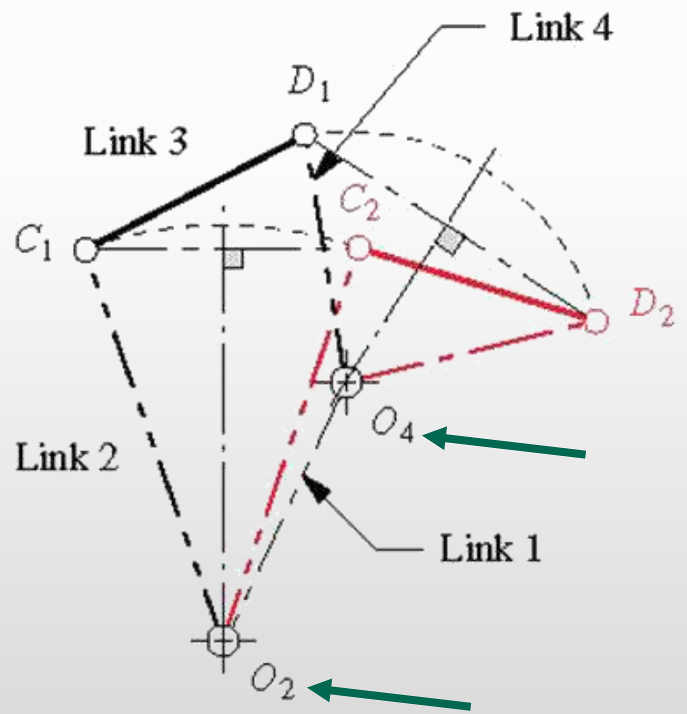
Bisect line C_1C_2 and line D_1D_2 and extend their perpendicular bisectors in convenient directions. The rotopole will not be used in this solution.



Geometric Linkage Synthesis – Case 3

2-Position Moving Pivots Motion – Step 3

Select any convenient point on each bisector as the fixed pivots O_2 and O_4 , respectively.

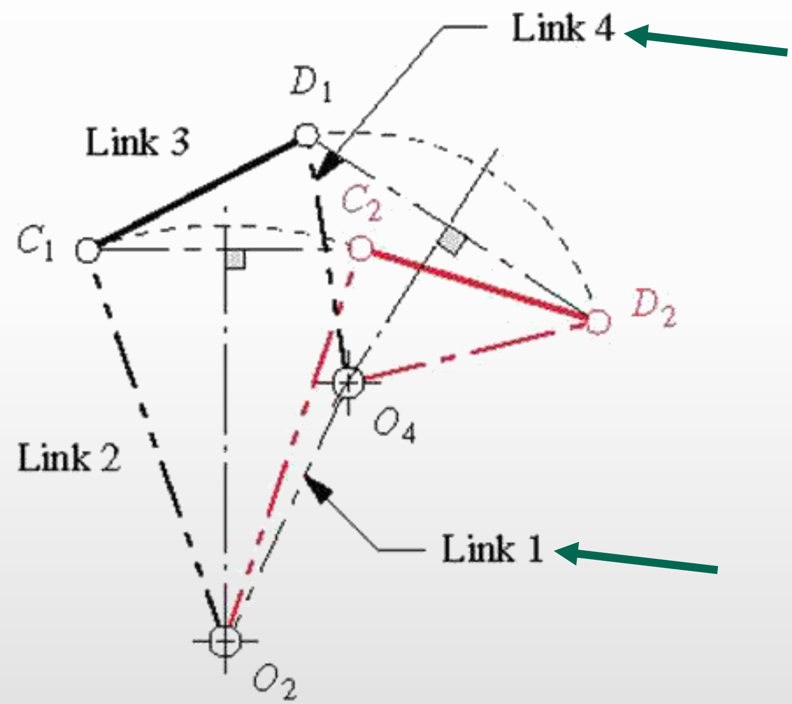


Geometric Linkage Synthesis – Case 3
2-Position Moving Pivots Motion – Steps 4-6

Step 4:
Connect O_2 with C_1 and call it link 2.
Connect O_4 with D_1 and call it link 4.

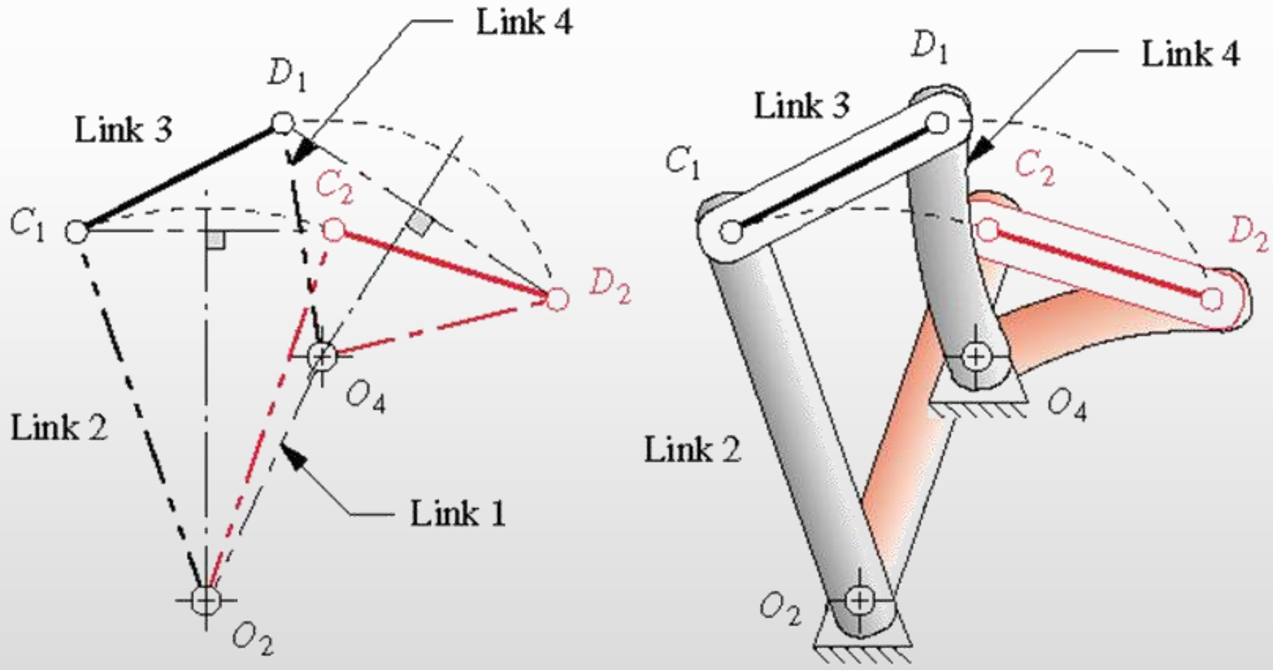
Step 5:
Line C_1D_1 is link 3.
Line O_2O_4 is link 1.

Step 6:
Check the Grashof condition,
and repeat steps 3 to 6 if unsatisfied.



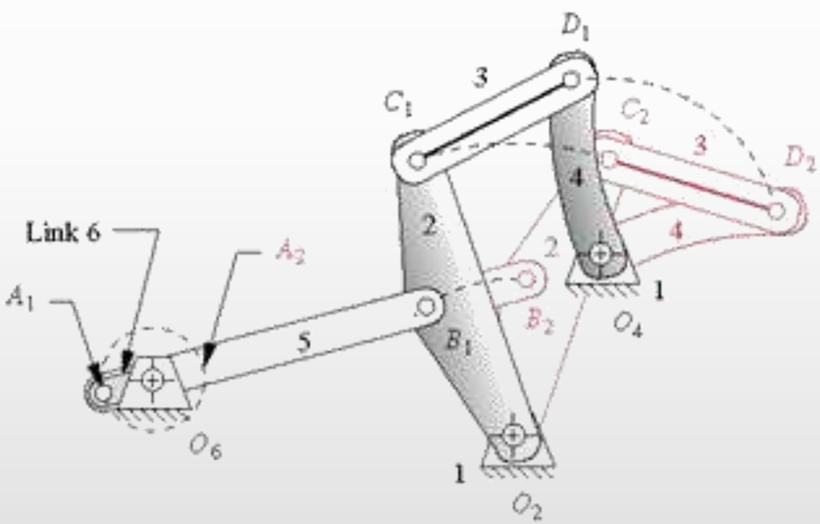
Geometric Linkage Synthesis – Case 3
2-Position Moving Pivots Motion – Step 7

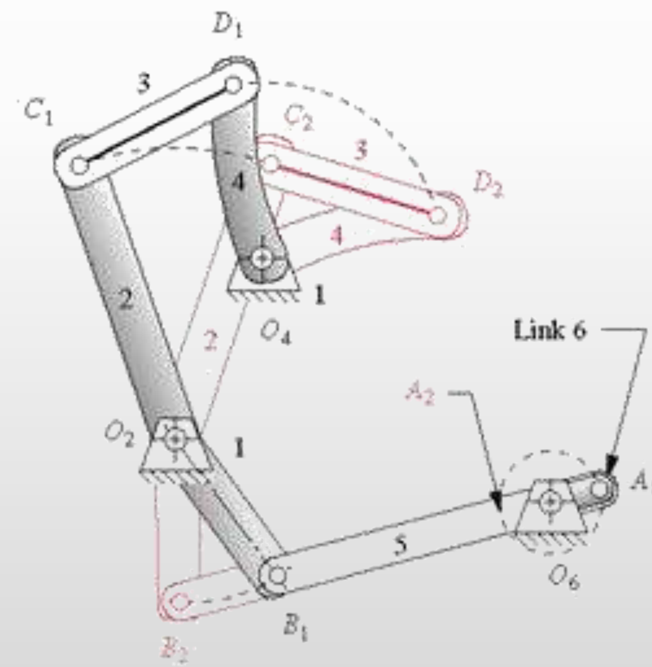
Make a model of the linkage and articulate it to check its function and its transmission angles



Combining Cases 1 and 3

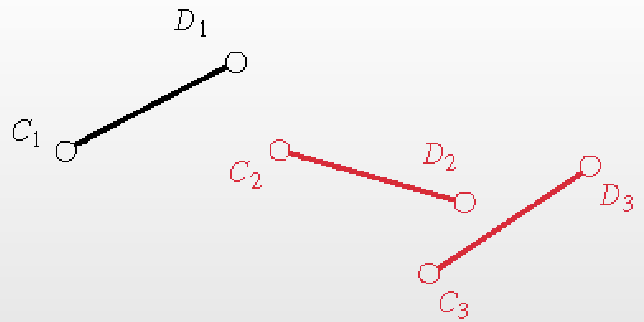
Design a dyad to control and limits the extremes of motion of the linkages in the previous example to its two design positions





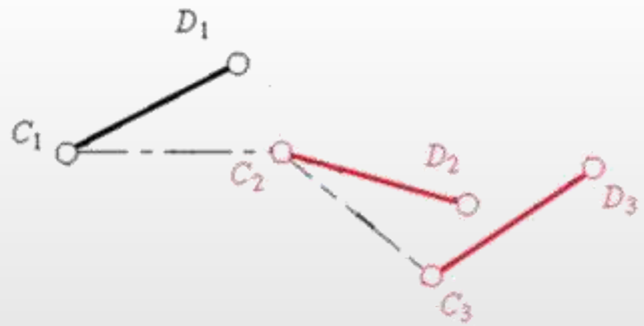
Geometric Linkage Synthesis – Case 4
3-Position Motion Generation

Design a four–bar Grashof linkage that moves C-D from the first position C_1D_1 to C_2D_2 and then to the position C_3D_3



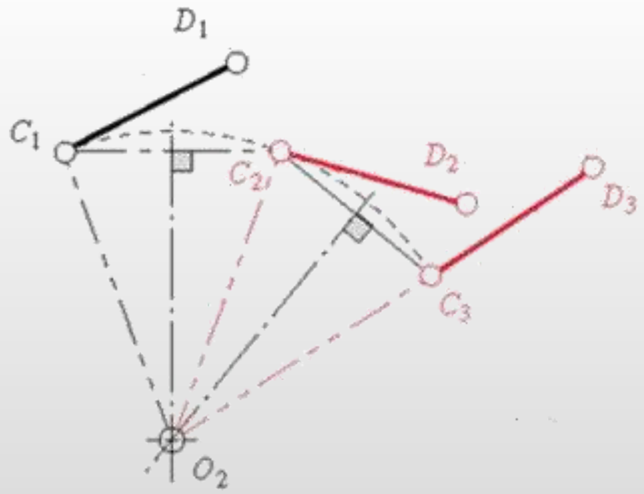
Geometric Linkage Synthesis – Case 4
3-Position Motion Generation – Step 1

Draw construction lines from point C_1 to C_2 and from C_2 to C_3



Geometric Linkage Synthesis – Case 4
3-Position Motion Generation – Step 2

Bisect line C_1C_2 and line C_2C_3 and extend their perpendicular bisector until they intersect. Label their intersection O_2.



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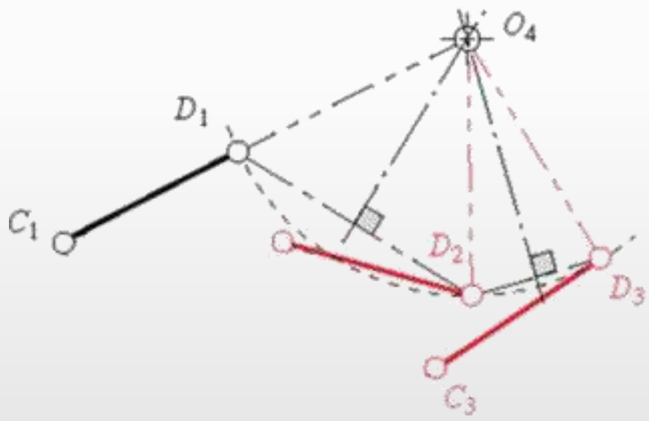
Geometric Linkage Synthesis – Case 4
3-Position Motion Generation – Step 3

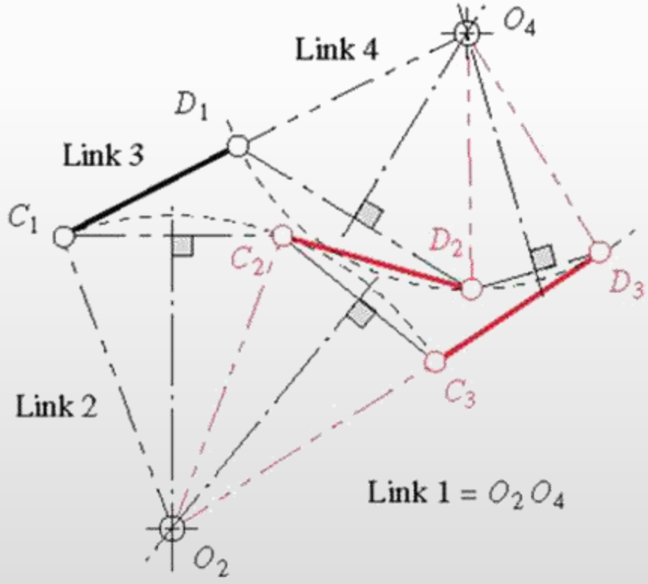
Repeat steps 2 and 3 for lines D_1D_2 and D_2D_3.
Label the intersection O_4.



Geometric Linkage Synthesis – Case 4

3-Position Motion Generation – Step 4

Connect O_2 with C_1 and call link 2.
Connect O_4 with D_1 and call link 4.
Line C_1D_1 is link 3.
Line O_2O_4 is link 1.



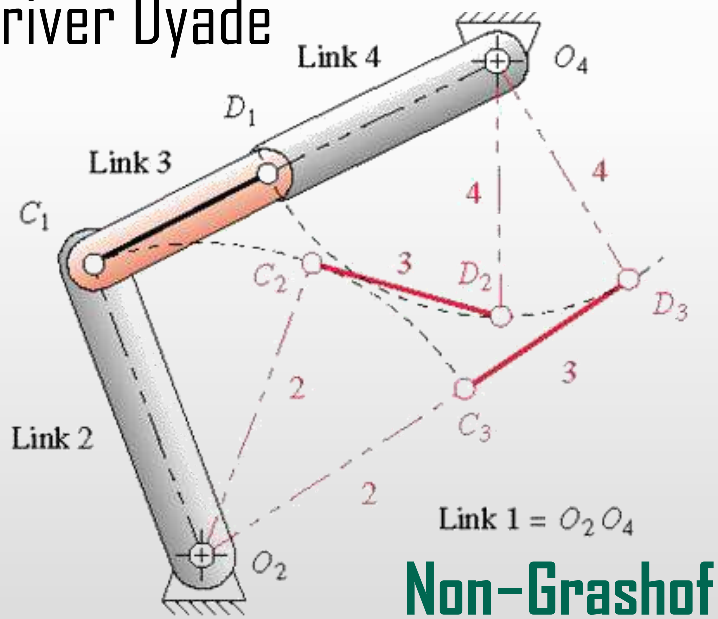
Geometric Linkage Synthesis – Case 4

3-Position Motion Generation – Steps 5-7

Step 5: Check the Grashof condition

Step 6: Construct Model and check toggle and transmission angles

Step 7: Construct Driver Dyade

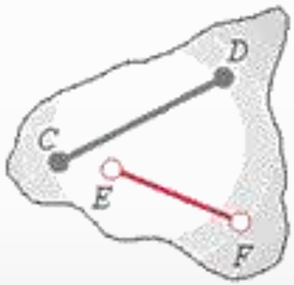


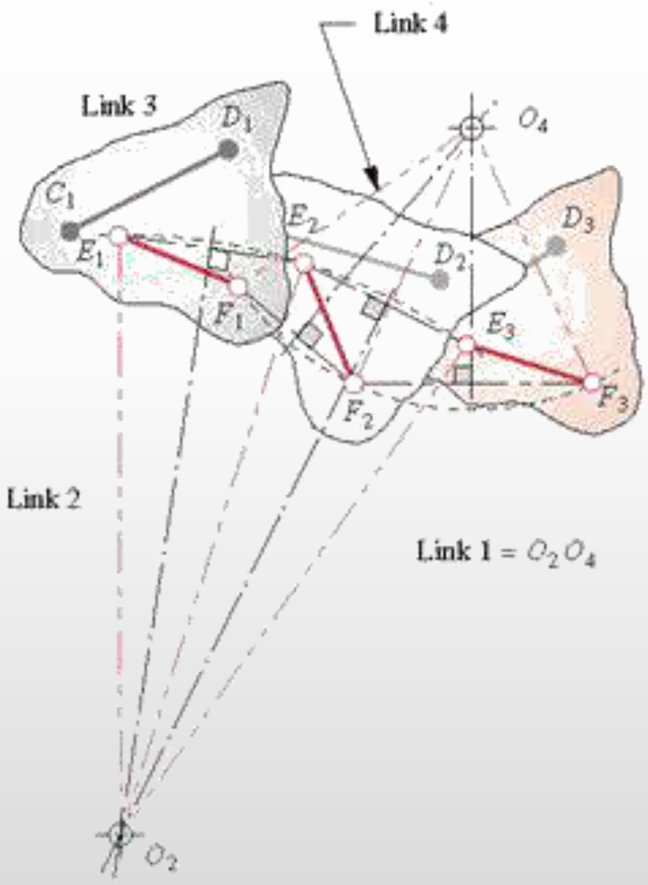
Non-Grashof

Geometric Linkage Synthesis – Case 5

3-Position Motion Generation – Alternate Attachment Points

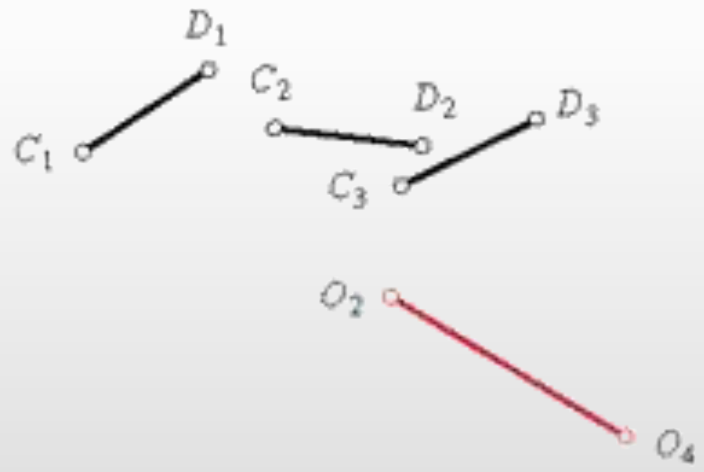
- Change the length of Link 3





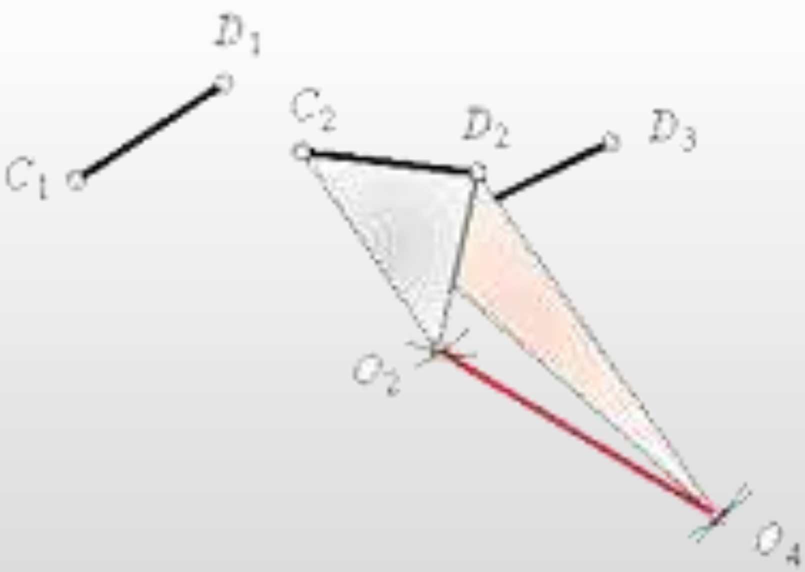
Geometric Linkage Synthesis – Case 6
3-Position Motion Generation with fixed pivots

Design a four bar linkage which move the link CD shown from position C_1D_1 to C_2D_2 and then to position C_3D_3.
Use specified fixed pivots O_2 and O_4.



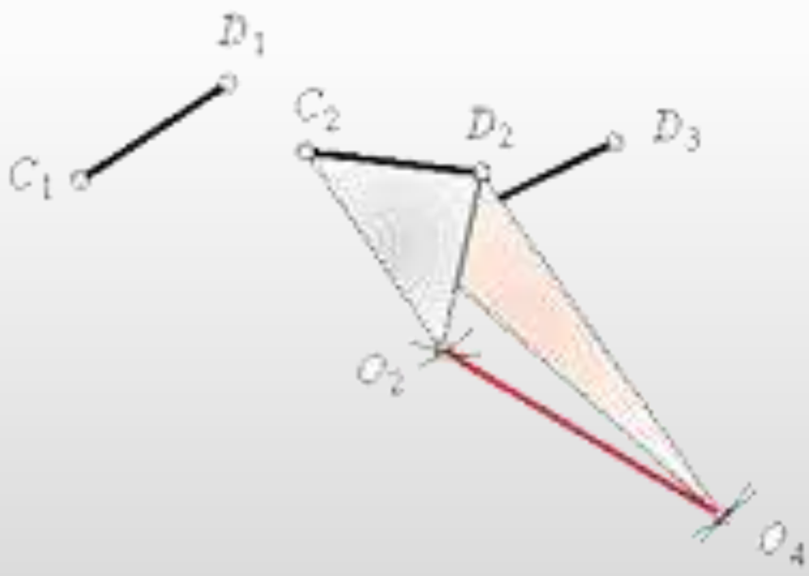
Geometric Linkage Synthesis – Case 6
3-Position Motion Generation with fixed pivots – Step 1a

Draw construction arc from point C_2 to O_2 and from D_2 to O_2 whose radii define the side of triangle $C_2O_2D_2$. This defines the relationship of the fixed pivot O_2 to the coupler line CD in the second coupler position.



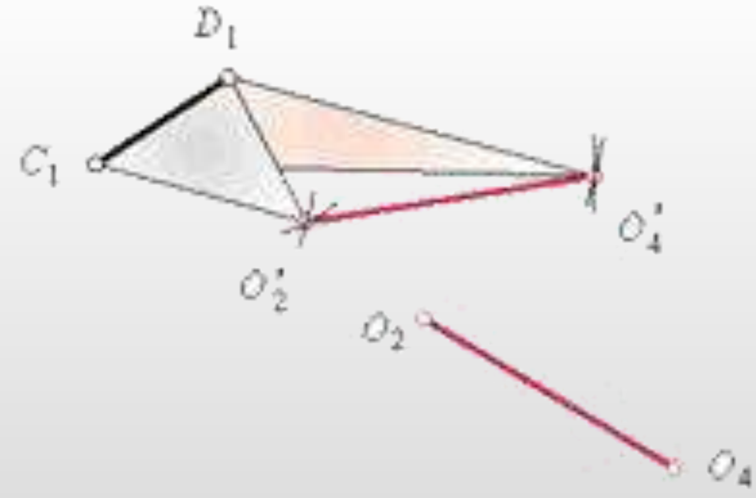
Geometric Linkage Synthesis – Case 6
3-Position Motion Generation with fixed pivots – Step 1b

Draw construction arc from point C_2 to O_4 and from D_2 to O_4 whose radii define the side of triangle $C_2O_4D_2$. This defines the relationship of the fixed pivot O_4 to the coupler line CD in the second coupler position.



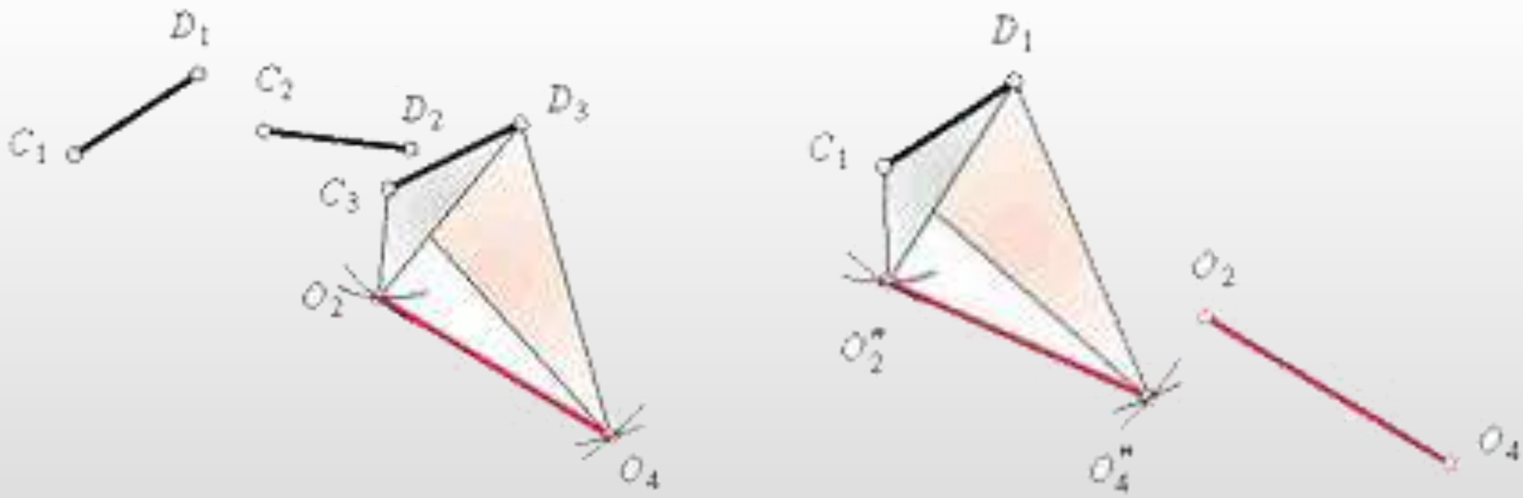
Geometric Linkage Synthesis – Case 6
3-Position Motion Generation with fixed pivots – Step 2

Now transfer this relationship back to the first coupler position C_1D_1 so that the ground plane position $O_2'O_4'$ bears the same relationship to C_1D_1 as O_2O_4 bore to the second coupler position C_2D_2. We have inverted the problem.



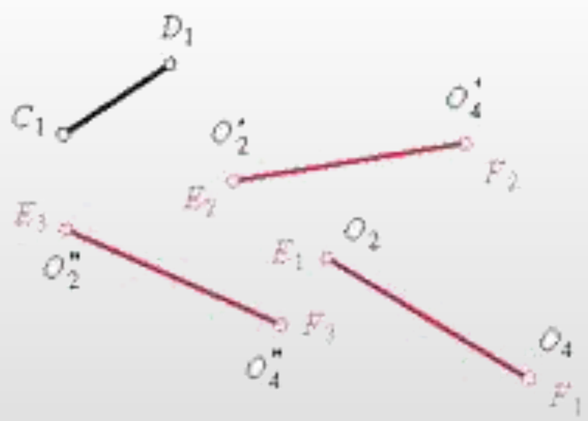
Geometric Linkage Synthesis – Case 6
3-Position Motion Generation with fixed pivots – Step 3

Repeat the process for the third coupler position as shown in the figure and transfer the third relative ground link position to the first, or reference, position



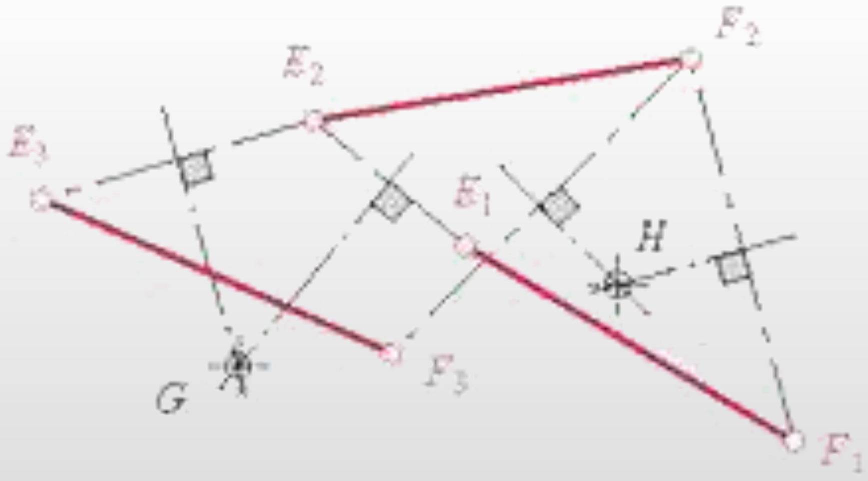
Geometric Linkage Synthesis – Case 6
3-Position Motion Generation with fixed pivots – Step 4

The three inverted position of the ground plane that correspond to the three desired coupler positions are labeled O_2O_4, $O_2'O_4'$, and $O_2''O_4''$ and have also been renamed E_1F_1, E_2F_2 and E_3F_3 as shown in the figure



Geometric Linkage Synthesis – Case 6
3-Position Motion Generation with fixed pivots – Step 5

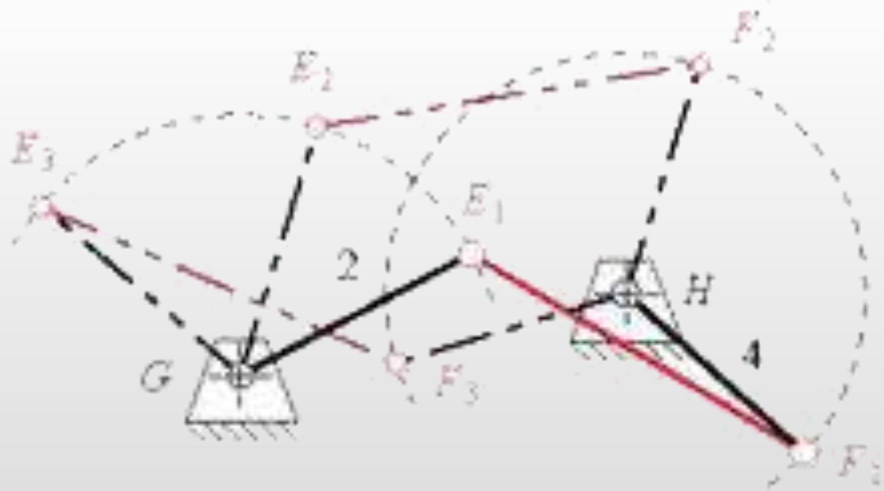
Find rotopoles G and H



Geometric Linkage Synthesis – Case 6

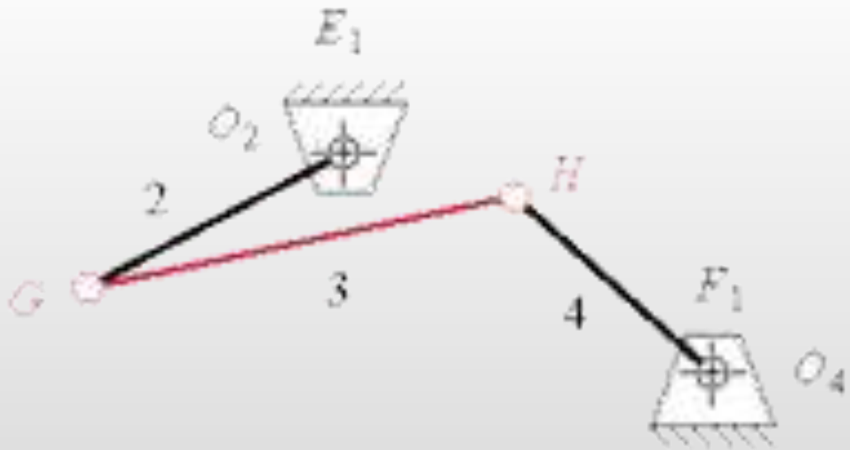
3-Position Motion Generation with fixed pivots – Step 6

Connect G with E_1 and call it link 2.
Connect H with F_1 and call it link 4.
Line E_1F_1 is the coupler, link 3.
Line GH is the ground link 1.



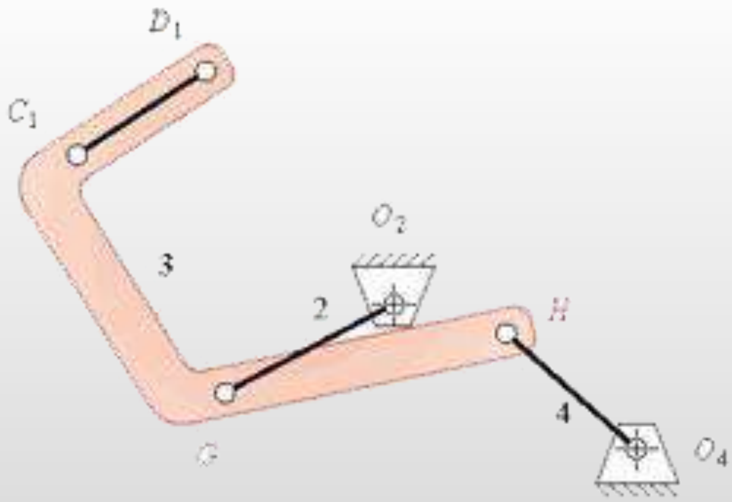
Geometric Linkage Synthesis – Case 6
3-Position Motion Generation with fixed pivots – Step 7

The figure shows the re-inversion of the linkage in which points G and H are now the moving pivots on the coupler and E_1F_1 has resumed its real identity as ground link O_2O_4



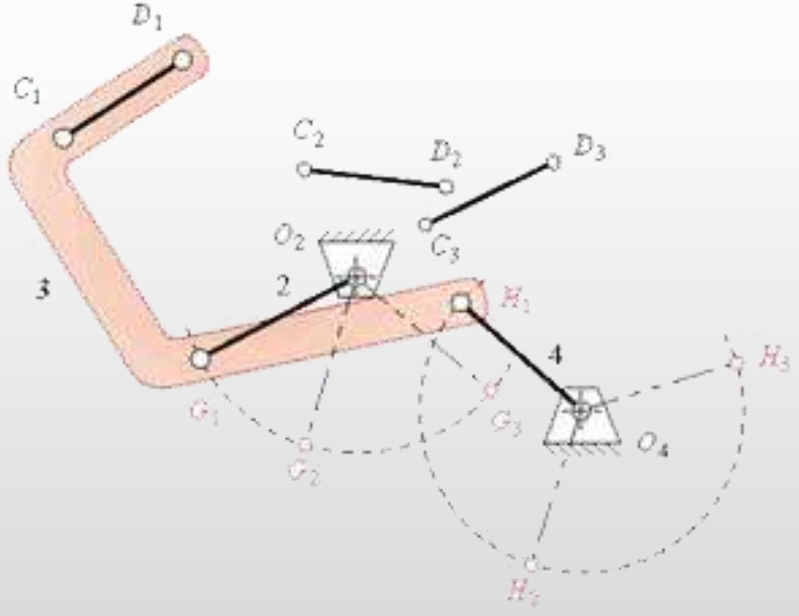
Geometric Linkage Synthesis – Case 6
3-Position Motion Generation with fixed pivots – Step 8a

Reintroduce the original line C_1D_1 in its correct relationship to line O_2O_4 at the initial position as shown in the original example. This form the coupler plane and defines a minimal shape of link 3.



Geometric Linkage Synthesis – Case 6
3-Position Motion Generation with fixed pivots – Step 8b

The angular motions required to reach the second and third position of line CD shown in the figure are the same as those defined in figure b for the linkage inversion



Geometric Linkage Synthesis – Case 6
3-Position Motion Generation with fixed pivots – Step 9

Step 9: Check the Grashof condition

Step 10: Construct a model and check toggle and transmission